Beyond Numbers

How Abstract Math Illuminates Our World

Alice Patania, Ph.D.

Assistant Professor
Mathematics and Statistics
Vermont Complex Systems Center

University of Vermont

Thinking like a mathematician 101

Pure math

↓

Applied math

↓

Science

↓

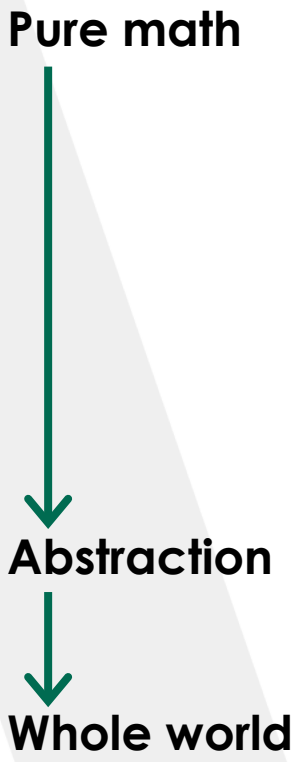
Engineering, Medicine,…

↓

Numerical world

University of Vermont

Thinking like a mathematician 101



University of Vermont

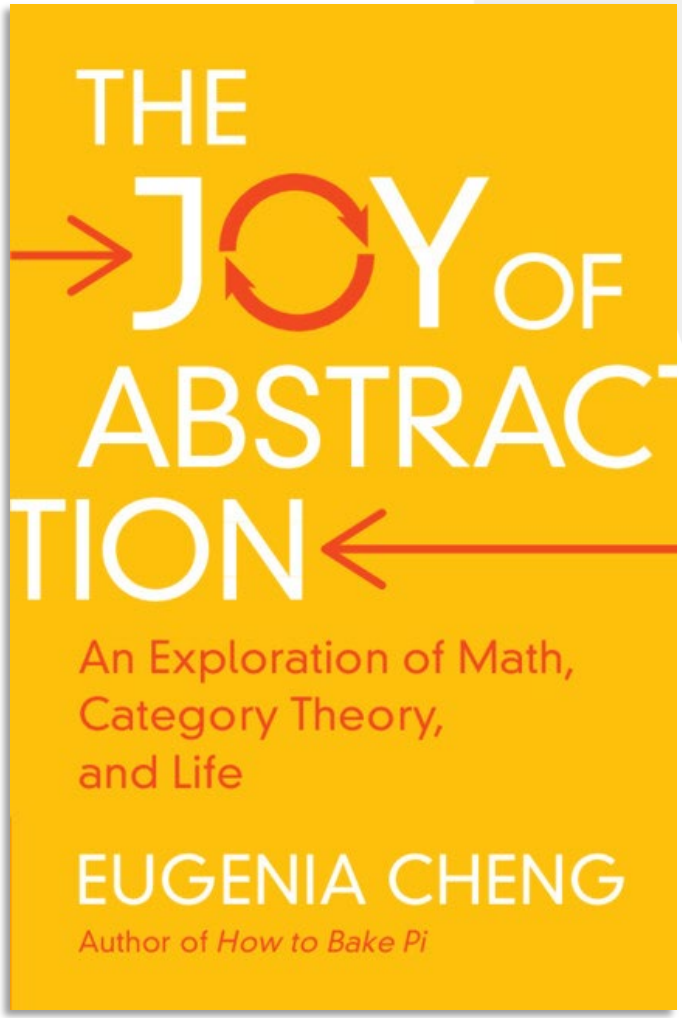
Concepts*

Patterns

Pivot

Relations

Abstraction



Patterns

Finding out what things have in common

University of Vermont

Patterns

Finding out what things have in common

3 things

University of Vermont

Patterns

Finding out what things have in common

3 pieces of fruit

University of Vermont

Patterns

Finding out what things have in common

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Pivot

Understanding your assumptions and learning from the differences

3 things

3 pieces of fruit

University of Vermont

NUMBER SYSTEMS

PERSIA
c.820

ALGEBRA

c.1730
MATHEMATICAL NOTATION

INDIA
c.628

FIRST ZERO **0**

EQUATION

$y=mx+c$

COUNTING

$e^{i\pi}=-1$

NEGATIVE NUMBERS

-8 π

CHINA
200 BCE

50,000 BCE

EGYPT
FIRST EQUATION
3000 BCE

GREECE
600-300 BCE

SPACES

ORIGINS

THE MAP OF MATHEMATICS

FOUNDATIONS

FUNDAMENTAL RULES

MATHEMATICAL LOGIC

$p \Rightarrow q$

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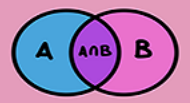

CONSISTENT SET OF AXIOMS?

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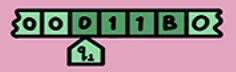
CATEGORY THEORY

THEORY OF COMPUTATION



$P \neq NP$?

COMPLEXITY THEORY

CARDINAL NUMBERS

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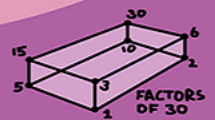
ARITHMETIC

$+ - \times \div$

COMBINATORICS

PARTITION THEORY

TREE



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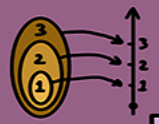

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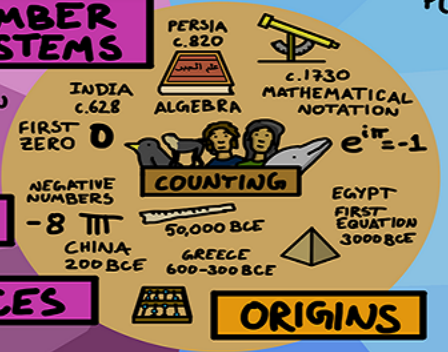


MÖBIUS STRIP

TOPOLOGY

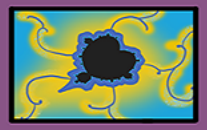
DIFFERENTIAL GEOMETRY

1 HOLE (GENUS 1)

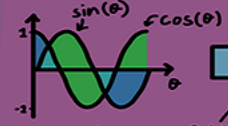
COMPLEX ANALYSIS

FRACTAL GEOMETRY

MANDELBROT SET



DYNAMICAL SYSTEMS

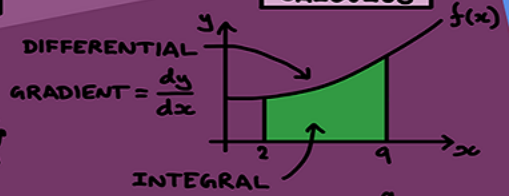


PYTHAGORAS

TRIGONOMETRY

GEOMETRY

CALCULUS



DIFFERENTIAL EQUATIONS

VECTOR CALCULUS

FLUID FLOW

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APPLIED MATHEMATICS

CRYPTOGRAPHY

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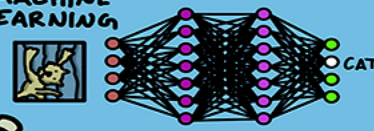

PROBABILITY

BAYES' RULE

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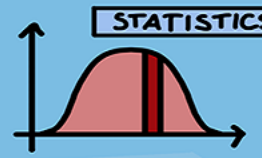
STATISTICS



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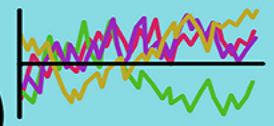
OPTIMIZATION

MATHEMATICAL FINANCE



ECONOMICS

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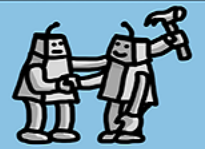
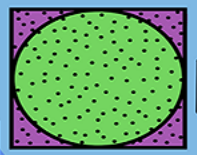
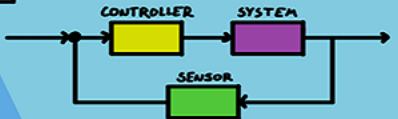


NUMERICAL ANALYSIS



ENGINEERING

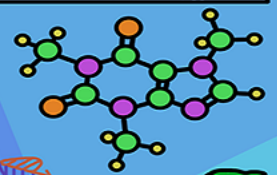


CONTROL THEORY

MATHEMATICAL PHYSICS

THEORETICAL PHYSICS

MATHEMATICAL CHEMISTRY

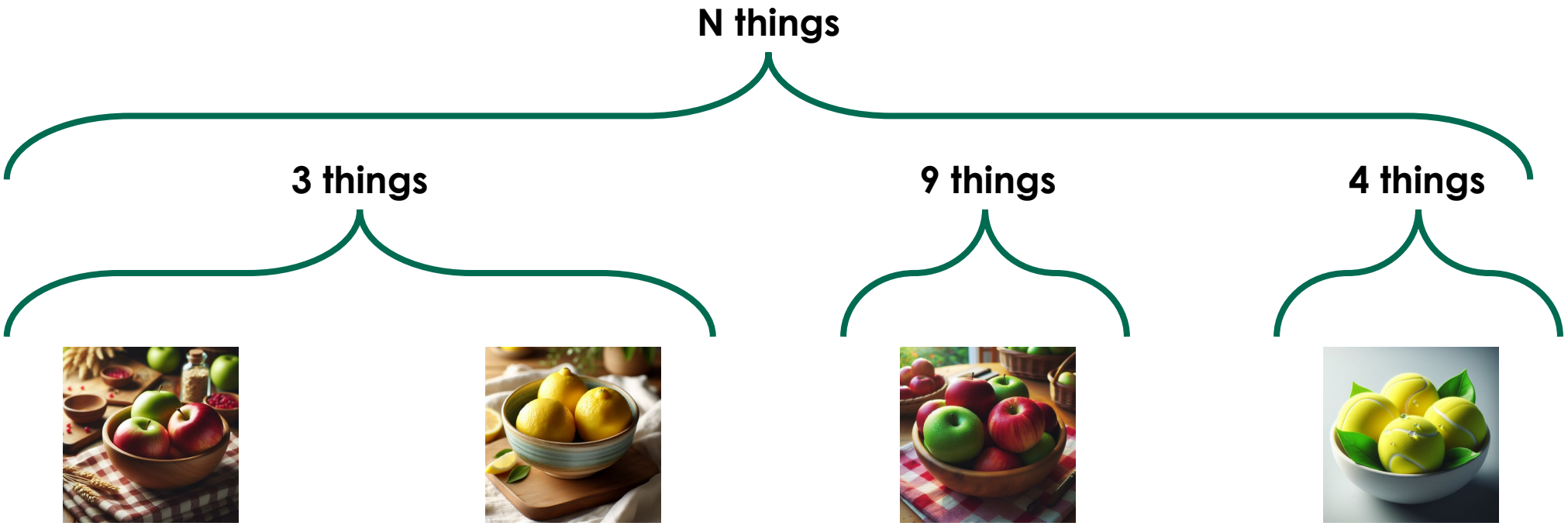


BIOMATHEMATICS

YOUTUBE: THE MAP OF MATHEMATICS

Patterns

Finding out what things have in common



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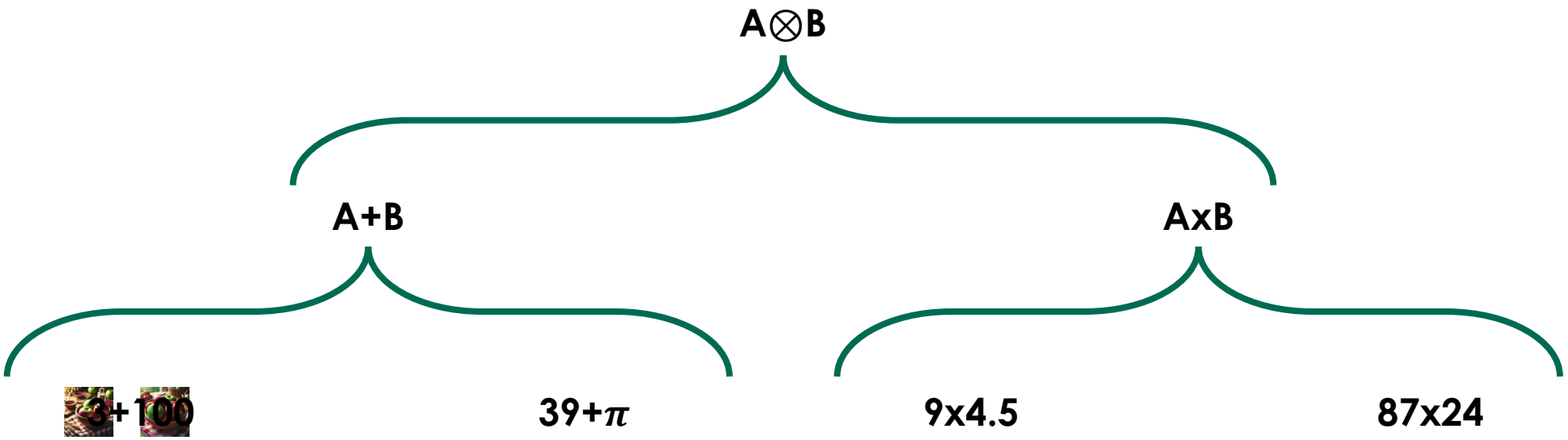

University of Vermont

Mathematician

Operation 3 Birthday cakes

S	M	T	W	T	F	S
28	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31	1	2	3
4	5	6	7	8	9	10

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Operation 3 Birthday cakes

There are 365 days between Sunday 2 February 1997 and 2 February 1998.

365 days = 52 weeks + 1 day

This means that 2 February 1998 will be a **Monday**!

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Thinking like a mathematician 101

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Engineering, Medicine,…

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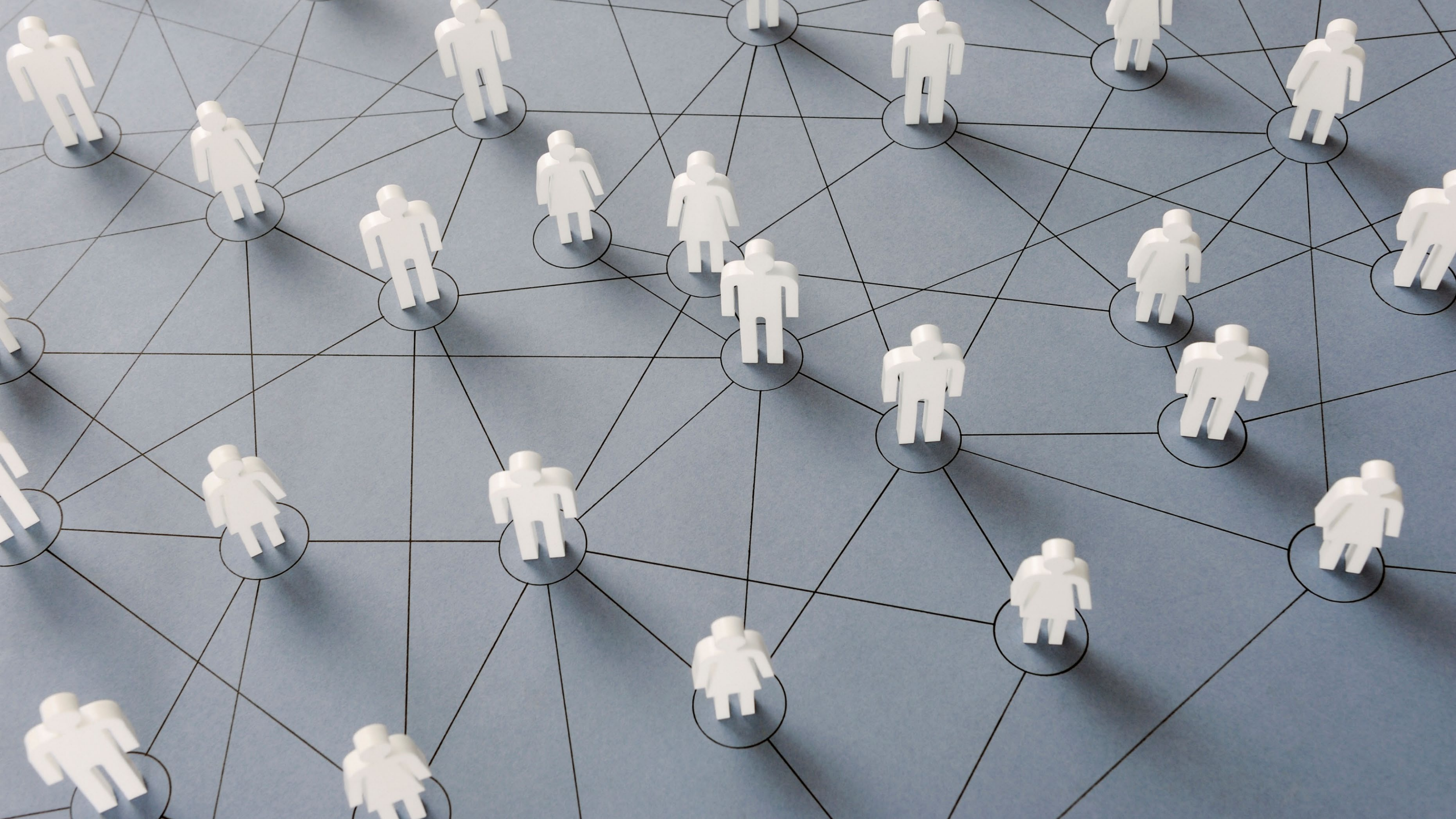
University of Vermont



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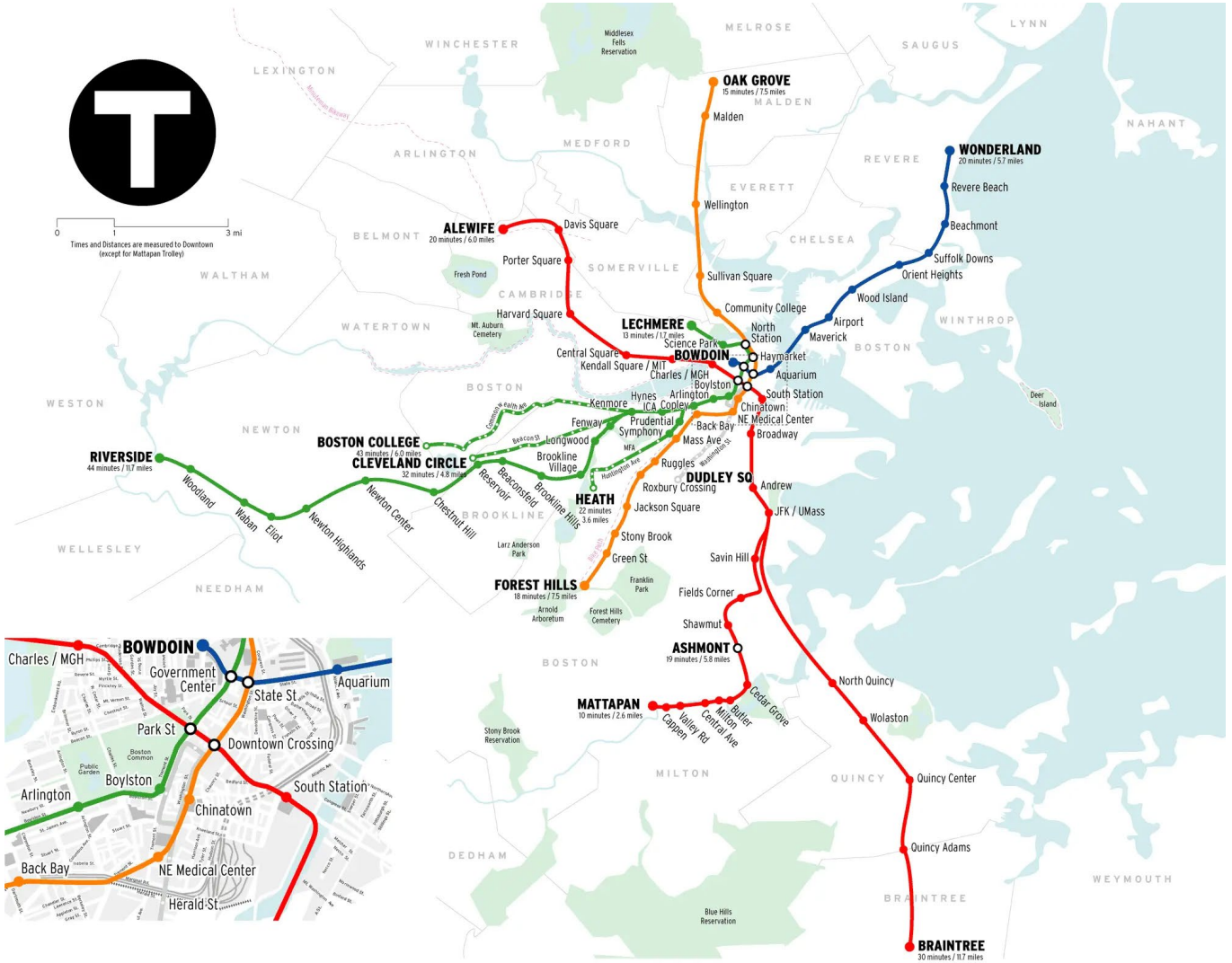
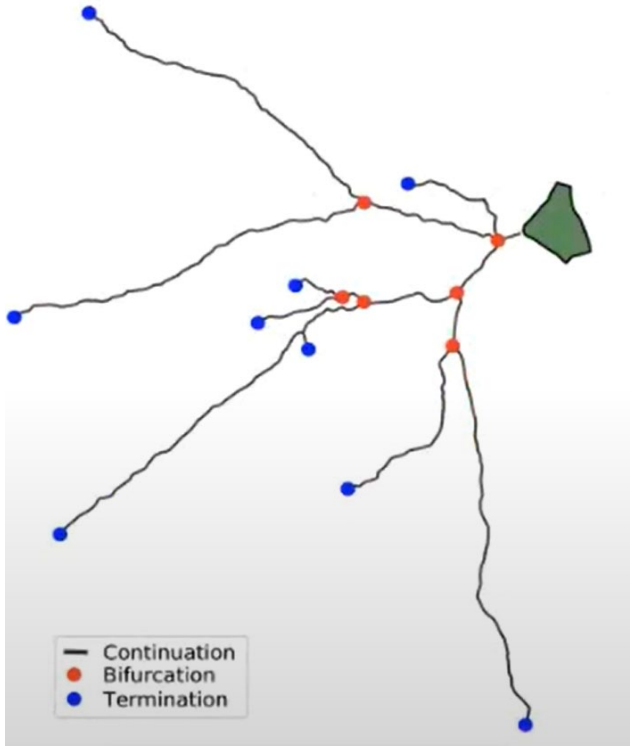
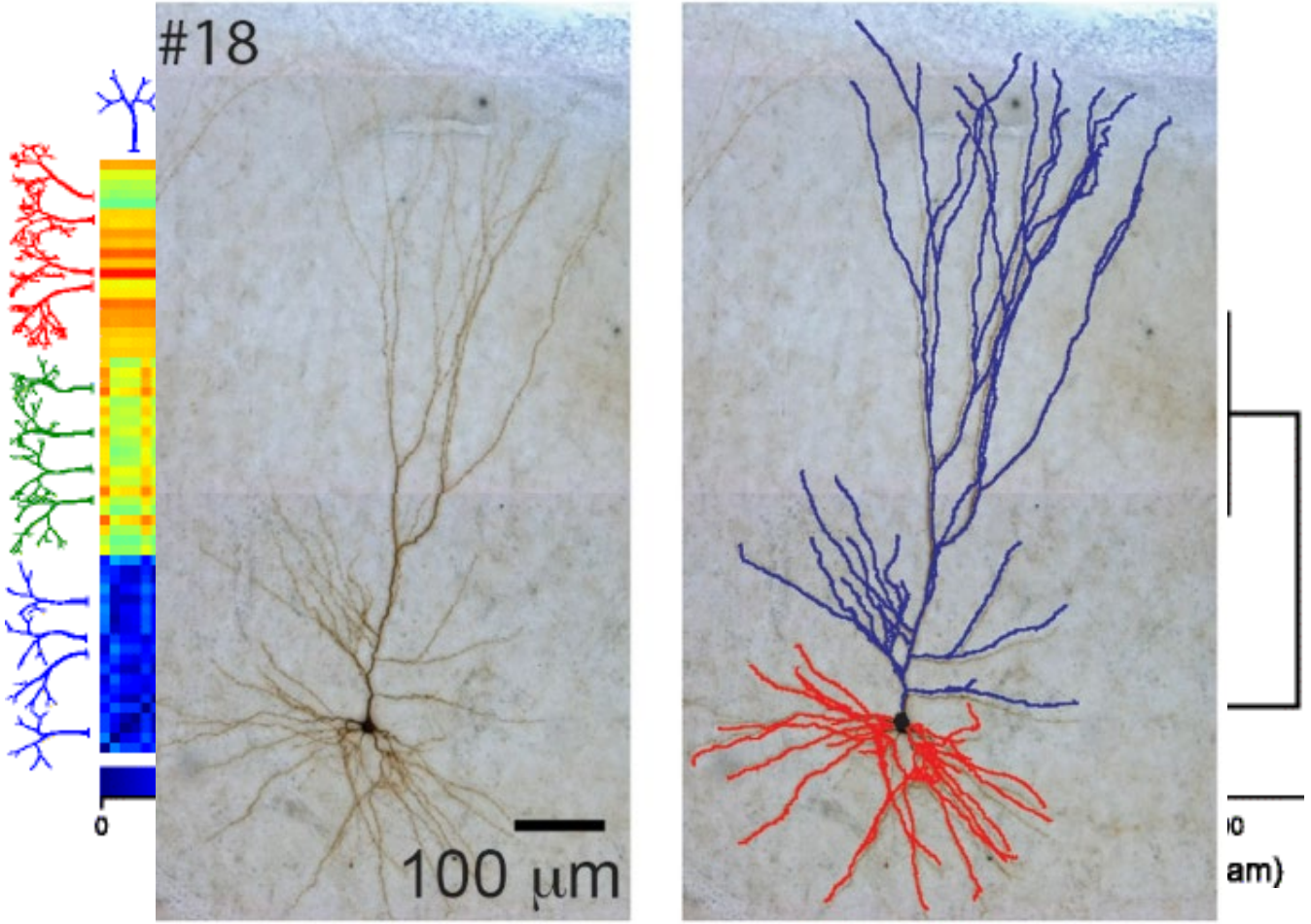


University of Vermont

Relations+Patterns

Classifying human pyramidal neurons





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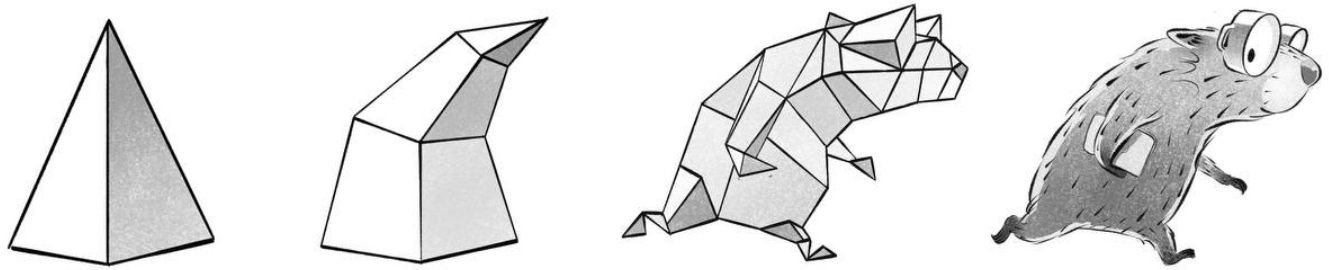
What is my job?

I am a computational topologist



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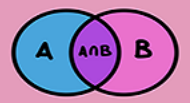

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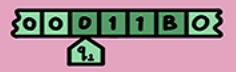

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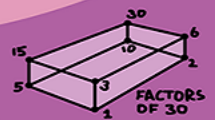

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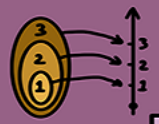

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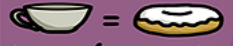

1 HOLE (GENUS 1)

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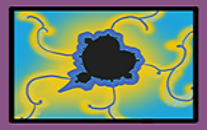

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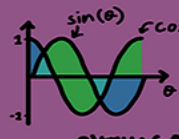

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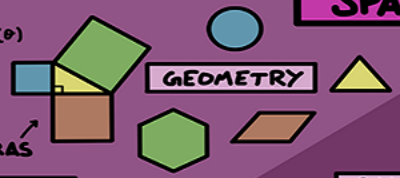
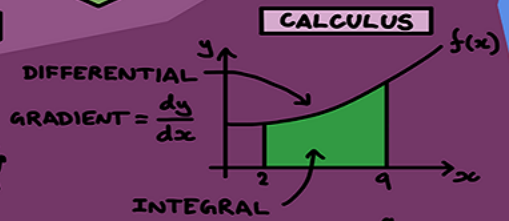


GEOMETRY

VECTOR CALCULUS

CALCULUS



DIFFERENTIAL

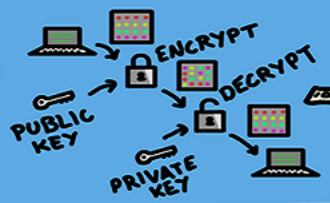
GRADIENT = $\frac{dy}{dx}$

INTEGRAL

$\text{AREA} = \int_2^9 f(x)\,dx$

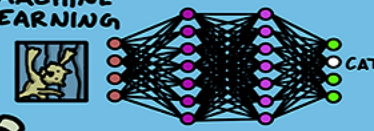
DIFFERENTIAL EQUATIONS

CRYPTOGRAPHY



COMPUTER SCIENCE

MACHINE LEARNING



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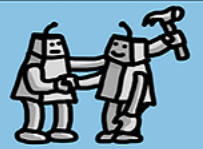
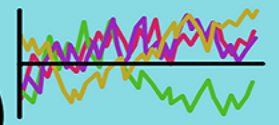
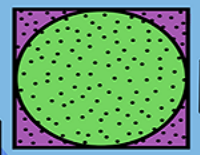


MATHEMATICAL FINANCE



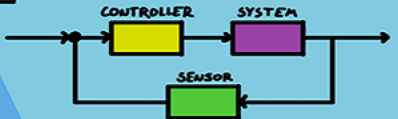
ECONOMICS

APPLIED MATHEMATICS



NUMERICAL ANALYSIS

ENGINEERING



CONTROL THEORY

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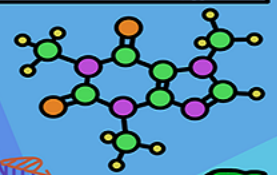
THEORETICAL PHYSICS

MATHEMATICAL CHEMISTRY



BIOMATHEMATICS

BY DOMINIC WALLIMAN © 2017

YOUTUBE: THE MAP OF MATHEMATICS

What is Topology?

University of Vermont

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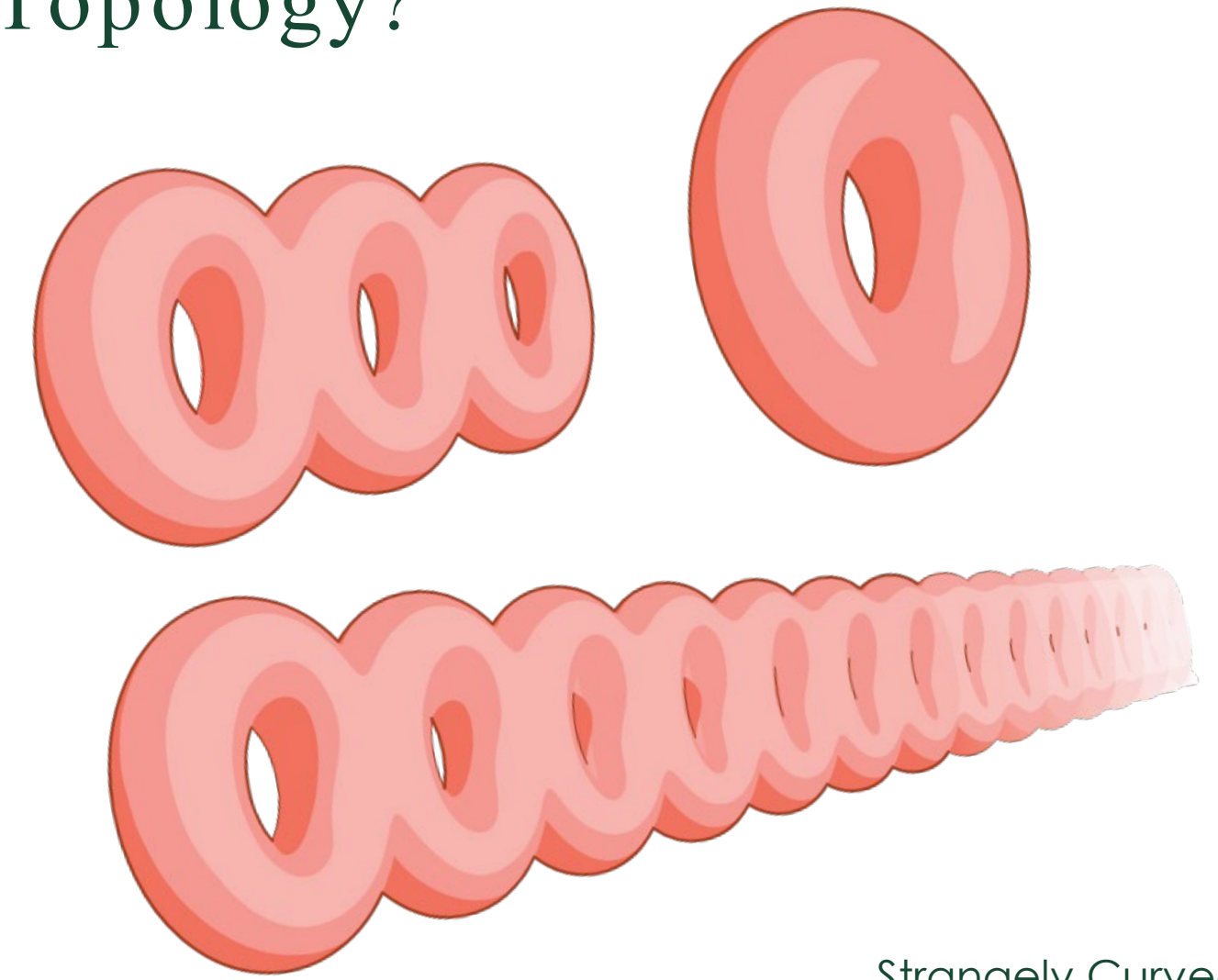
What is Topology?



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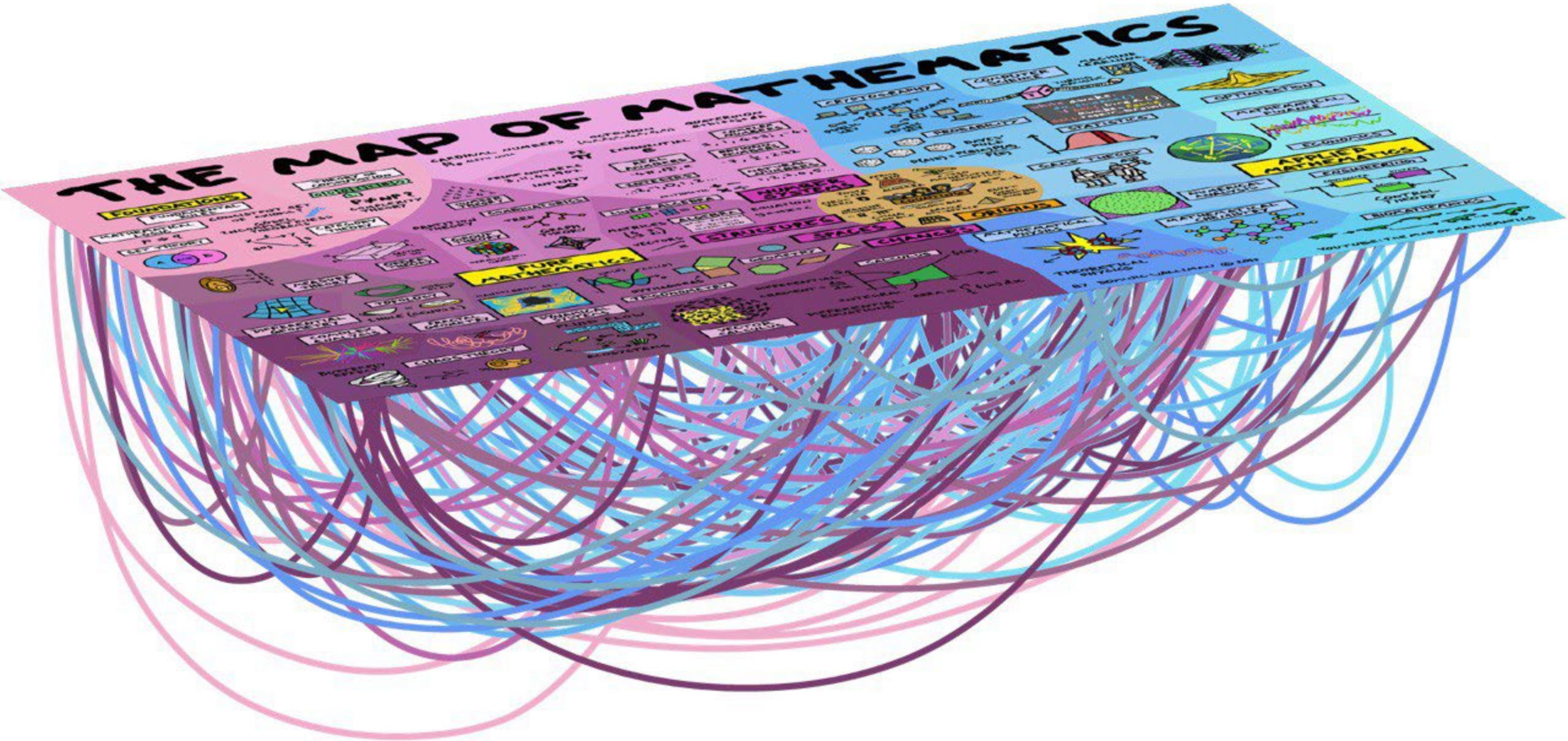
Strangely Curved Shapes Break 50-Year-Old Geometry Conjecture | Quanta Magazine


THE MAP OF MATHEMATICS
FOUNDATIONS
PURE MATHEMATICS
NUMBER SYSTEMS
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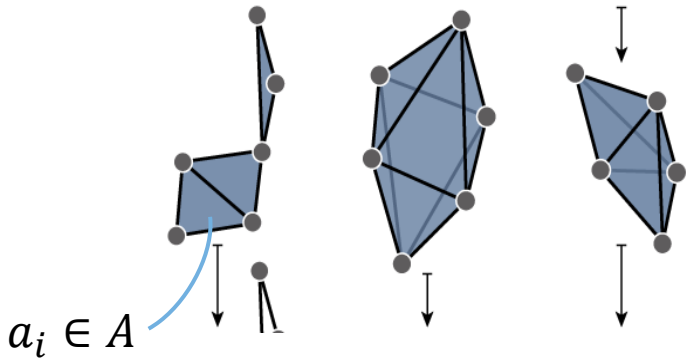
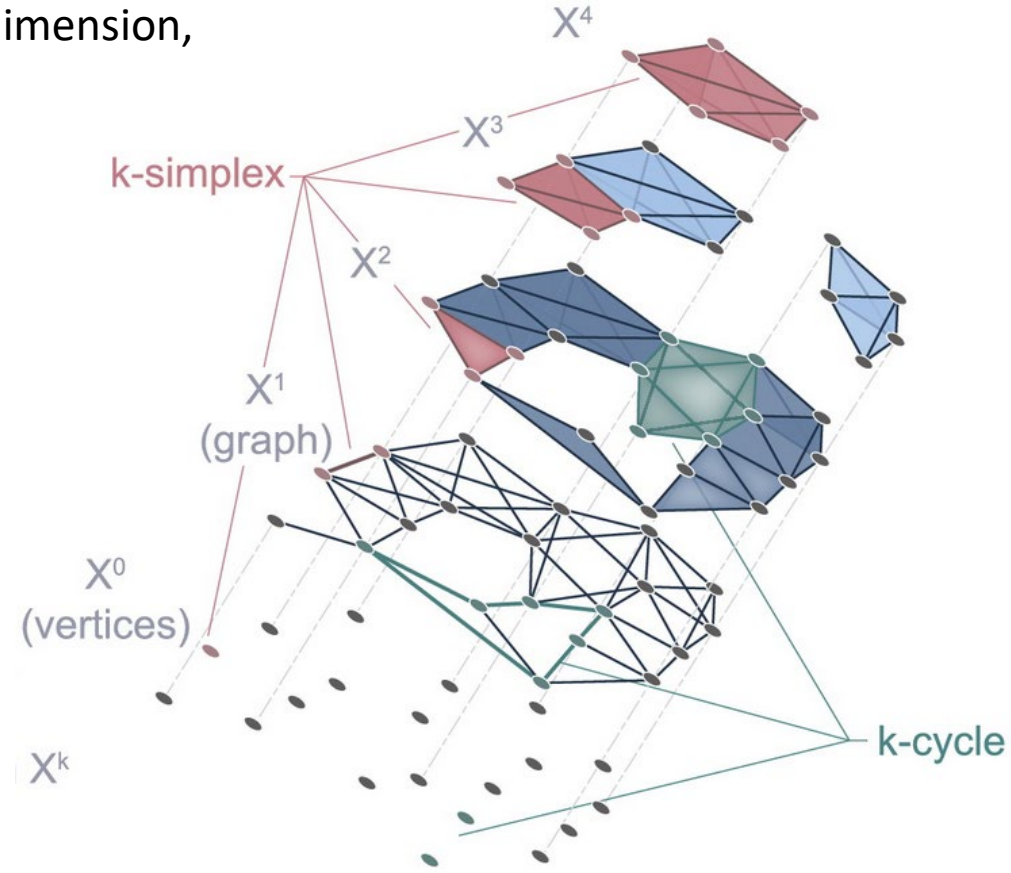
SPACES
CHANGE
APPLIED MATHEMATICS

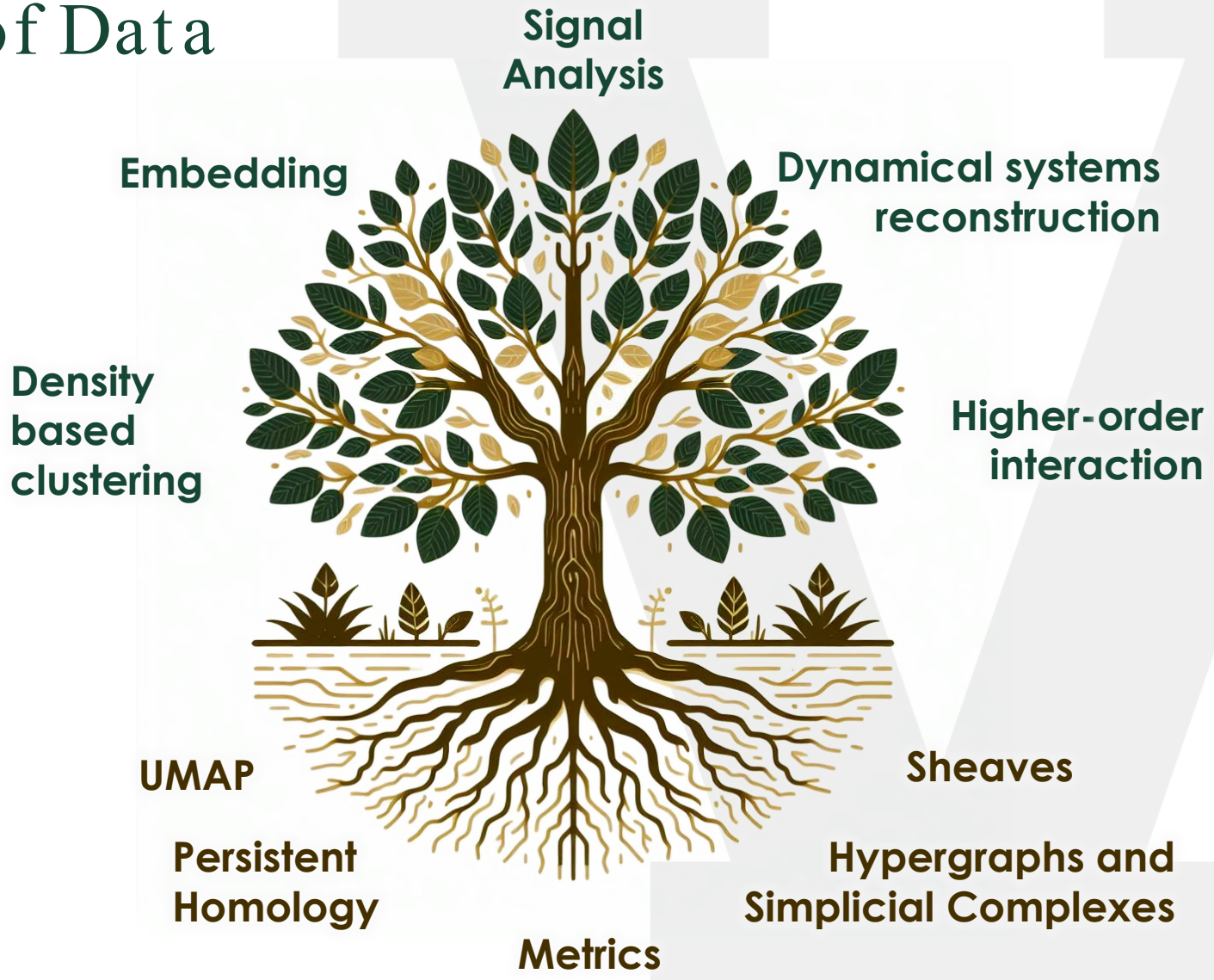
Encoding complexity

A ***k*-chain** is a formal sum of ordered simplices of the same dimension, $c = \sum a_i \sigma_i$ with $a_i \in A$ an abelian group.
The set of k-chains of Σ, C_k , is called the k-chain group of Σ.





Finding the Shape of Data



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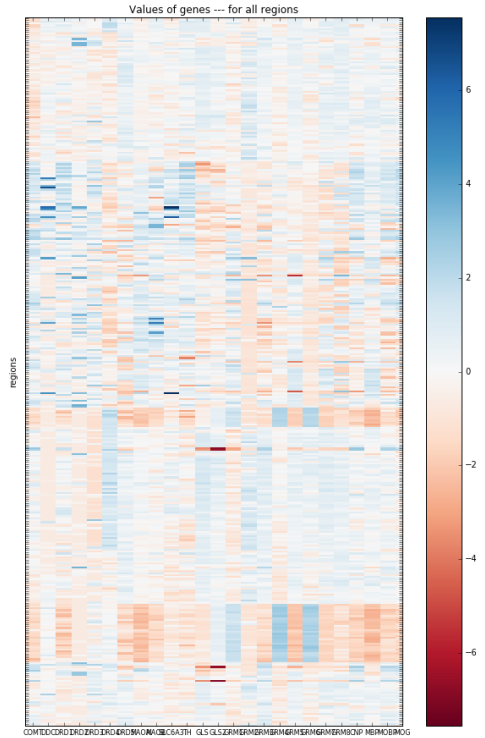
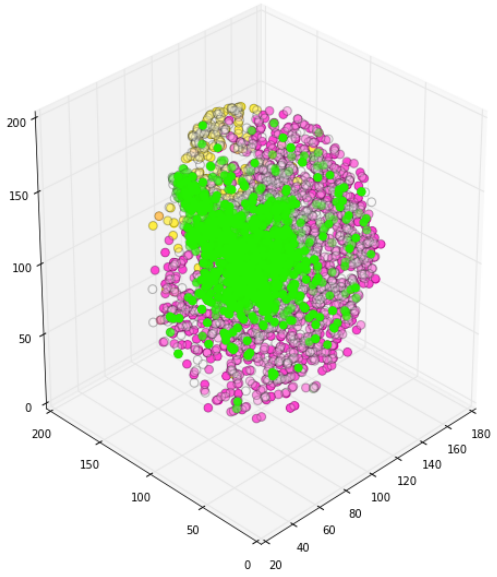
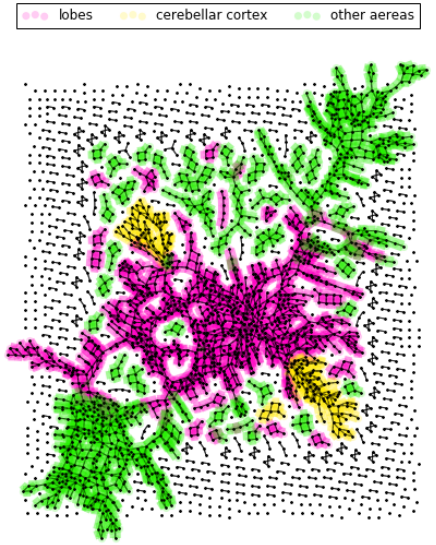
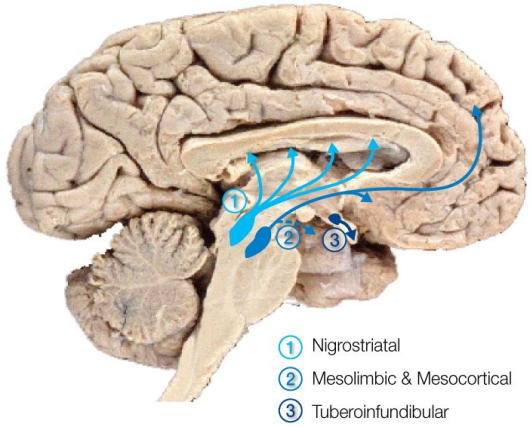
Brain Data

Brain cell gene-expression network recovers dopamine pathway









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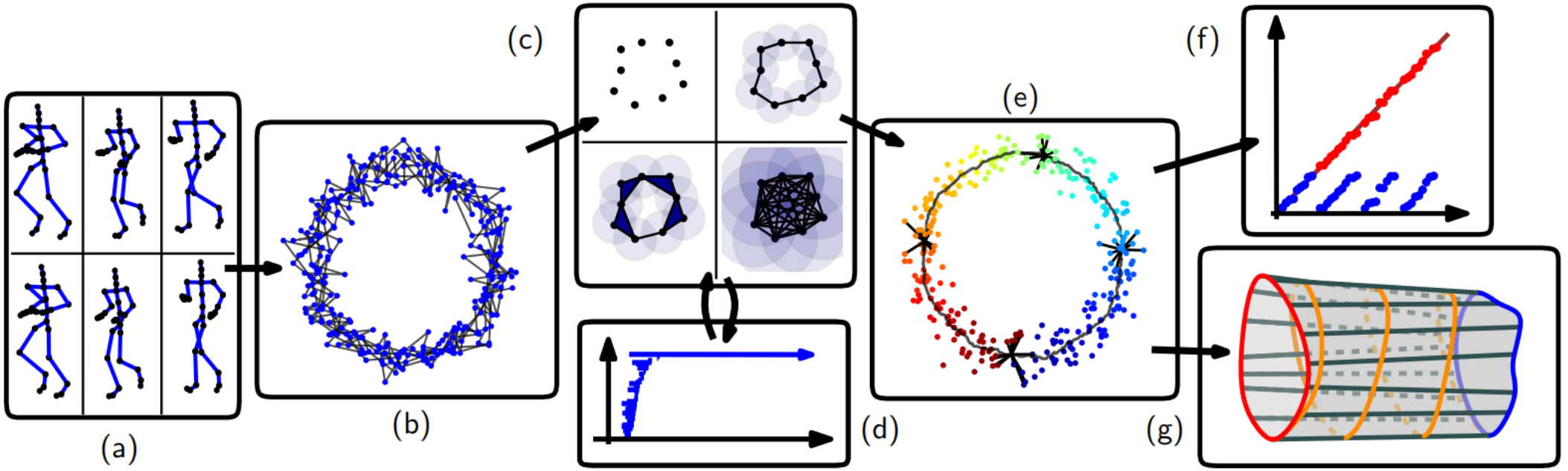
Cohomological Learning of Periodic Motion



Vejdemo-Johansson, M., Pokorny, F.T., Skraba, P. and Kragic, D., 2015. Cohomological learning of periodic motion. Applicable algebra in engineering, communication and computing, 26(1), pp.5-26.

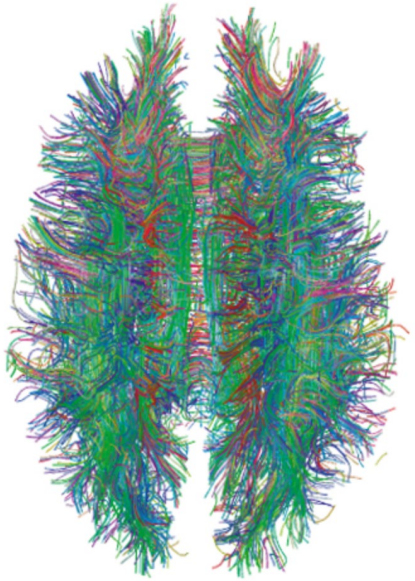
Berwald, J., Gidea, M. and Vejdemo-Johansson, M., 2014. Automatic Recognition and Tagging of Topologically Different Regimes in Dynamical Systems. Discontinuity, Nonlinearity, and Complexity, 3(4), pp.413-426.

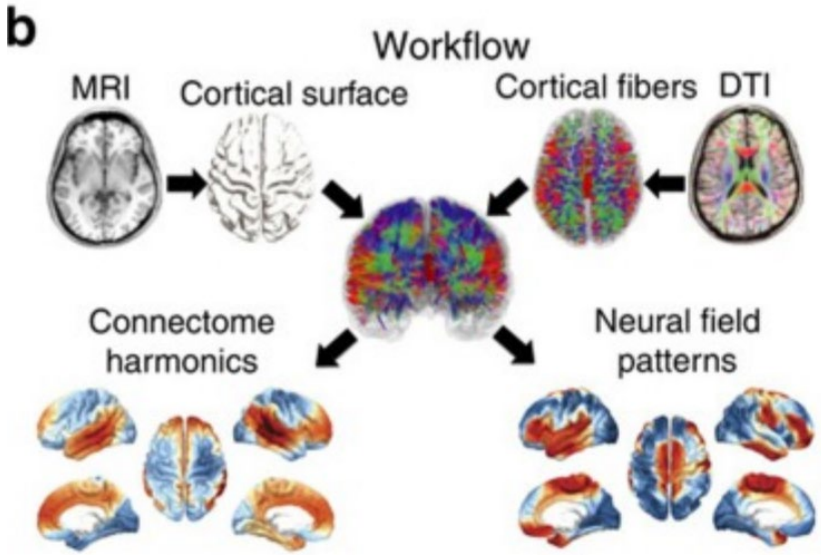
Decoding Parallel Processing in the Brain using Connectome Eigenfunctions

Work in Progress with T. Timofeyev

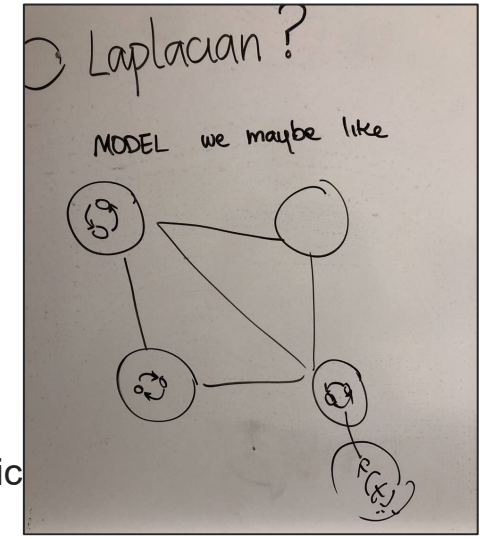
In mathematics, **Hodge theory** is a method for studying the cohomology groups of a smooth manifold M using partial differential equations.
Every **cohomology class** has a canonical representative (called **harmonic**), a differential form that vanishes under the Laplacian operator.

Wikipedia





Create a model that incorporates the harmonics of the connectome graph



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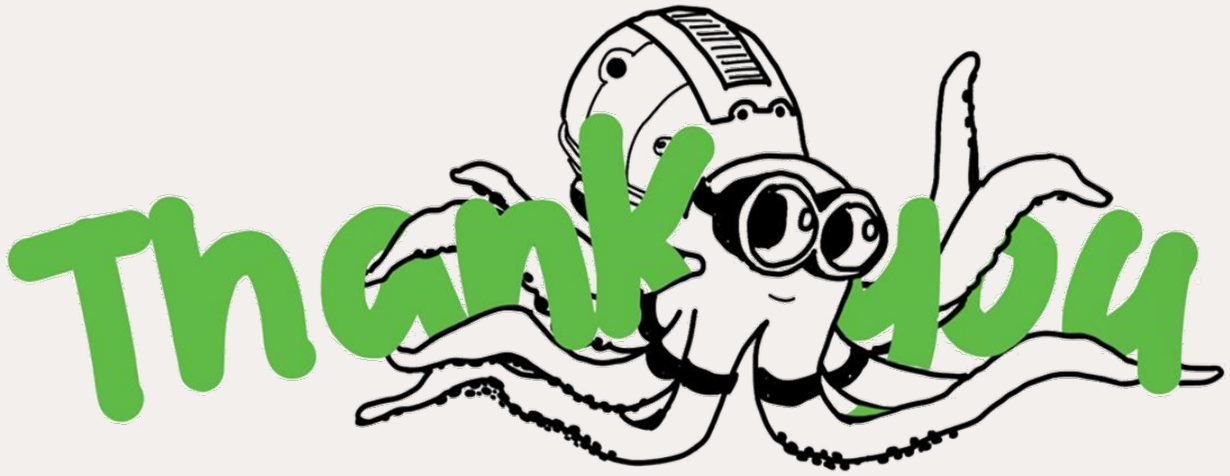
Atasoy S. et al. 2016. Human brain networks function in connectome-specific harmonic
Nat. comm. 7(1), p.10340.



Website alpatania.github.io
e-mail apatania@uvm.edu

Collaborators

Neuroscience

Liana Apostolova	Indiana University – School of Medicine
Jingwen Yan	IUPUI - School of Informatics and Computing
Olaf Sporns	Indiana University – Psychology and Brain Science
Apoorva Bharthur Sanjay	Indiana University – School of Medicine
Thomas Varley	Indiana
Vanessa Denny	Indiana University

Structure

Elizabeth Bruch	University of Michigan
Paul Expert	Imperial College London
Jean-Gabriel Young	University of Vermont
Antoine Allard	Universitee Laval
Giovanni Petri	ISI Foundation
Francesco Vaccarino	Politecnico di Torino

Dynamics

Carina Curto	Pennsylvania State University
Felicia Burtscher	University of Luxemburg
Stefania Ebli	EPFL Lausanne
Daniela Egas	EPFL Lausanne
Katie Morrison	University of Northern Colorado
Nicole Sanderson	Lawrence Berkeley National Laboratory
Tobias Timofeyev	University of Vermont *PhD Student

python GI

Nicholas Landry	University of Vermont
Leo Torres	MPI Leipzig
Maxime Lucas	CENTAI
Iacopo Iacopini	CEU – JSMF Fellow
Giovanni Petri	CENTAI
Alice Schwarze	Dartmouth

AATRN Applied Algebraic Topology Research Network

NIH National Institutes of Health

R21 - **Integrative Predictive Modeling of Alzheimer's Disease**
NIH Exploratory/Developmental Research Grant